

Short Elucidating Note 111: How to link the structural paradigm falsification theorem $K_i = L_j$ with existing true sustainability index theory as invalidation tools? What are the main implications of achieving this link?

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ABSTRACT: The theory of the structural paradigm falsification indicates that when conjunctural systems or systems that claim to have the inherently conjunctural form $K_i = L_j$ are placed under unilateral drivers (e.g. Maximization), then structural sustainability gaps are created. The theory of true sustainability indices tells us that when sustainability ratios of forms such as $S1 = K_i / L_j$ or $S2 = L_j / K_i$ or $S3 = PK_i / NK_i$ or $S4 = PL_j / NL_j$ or $S = PK_i + PL_j / NK_i + NL_j$ are subjected to unilateral transformation (e.g. Maximization), then again index sustainability gaps are created. The index sustainability gap is linked one to one to structural sustainability gaps, linking structural falsification/validation with sustainability index invalidation/validation. In other words, structural flaws go one to one with indices gaps in true sustainability thinking. The main goal of this paper is to show step by step how maximization thinking leads to the establishing a direct link between structural paradigm falsification theories with the true sustainability index invalidation theory, pointing out in the process the main implications related to the making of this link.

I. METHODOLOGY

First, the terminology used in this paper is given. Second, the nature of a conjunctural world, of structural falsification, and of the true sustainability index falsification is described in detail. Third, the idea of structural system falsification and sustainability index system invalidation equivalencies is introduced. Fourth, the notion of the equivalence of the structural falsification ratio and the true sustainability index ratio is shared. Fifth, the General Structural Index Consistent Equivalence Condition and the General Structural–Index Inconsistency Equivalence Condition are highlighted. And sixth, the main implications and conclusions of the paper are given.

II. TERMINOLOGY

K = Component K	K_i = Component K_i
L = Component L	L_j = Component L_j
SG = Sustainability gap	MAX(K) = Maximization of component K
TSSI = True system sustainability index component K_i	SSI_{K_i} = System sustainability index for component K_i
$SSIL_j$ = System sustainability index for component L_j	P = Characteristics present
PK_i = Characteristic K_i present	N = number of desirable characteristics
PL_j = Characteristic L_j present	N_{K_i} = number of desirable K_i characteristics
NL_j = number of desirable L_j characteristics	* = Optimality/optimization
STIR = Structural index ratio	MAX = Maximization
$STIR_{K_iL_j}$ = Structural index ratio for component K_i and L_j	TSSIR = True system sustainability index ratio
$TSSIR_{K_iL_j}$ = True system sustainability index ratio for K_i and L_j	GTSSI = General true sustainability index

GTSSI _{KiLj} = General true sustainability index for component Ki and Lj	NSC* = Not consistent with system optimality/optimization requirements
TSSI _{KiLj} = True system sustainability index for component Ki and Lj	SSI = System sustainability index

III. THE NATURE OF A CONJUNCTURAL WORLD

It has been pointed out that the nature of the world under empirical observation is conjunctural, and the structure of this world, including the one when models claim to have an intermittently conjunctural structure, can be stated as $K_i = L_j$ (Muñoz 2026), and this can be used to highlight step by step the nature of structural falsification and the nature of sustainability index invalidation, aspects that can later be linked to establish the view of structural falsification and sustainability index invalidation equivalence, where indices provide the numerical representation of structural flaws.

IV. THE NATURE OF STRUCTURAL FALSIFICATION

1) The two ways nature of structural consistency in conjunctural System: The process of two ways structural falsification can be represented as $K_i \leftrightarrow L_j$. This means that this process is a) a bi-directional relationship; b) a joint determination; c) a balanced condition $K_i = L_j$; d) a joint optimization type of adjustment with outcome $(K_i = L_j)^*$; e) a process that leads to no sustainability gaps ($SG = 0$); and f) a process where structural consistency is maintained or restored. And therefore, the above is the only structurally admissible representation of a conjunctural system under conjunctural consistency assumptions or within conjunctural true sustainability theory. Notice that when systems are treated conjuncturally the components are jointly determined and equalities stabilize coherently resulting in no sustainability gap creation; and therefore, there is no artificial inflation of structure.

2) The one way nature structural inconsistency in separable systems: If we assume or take conjunctural systems $K_i = L_j$ as separable, then we introduce structural inconsistencies when we apply unilateral pressures to either K_i or to L_j .

❖ Case A: Lj-driven system

If we apply unilateral pressure to L_j , we create a system led by L_j , which can be stated as $K_i \leftarrow L_j$. The characteristics of the L_j led system are a) L_j is the independent driver of K_i ; b) K_i becomes the dependent outcome; c) maximization of L_j ($MAX(L_j)$) breaks the system equality in the sense that it destabilizes the conjunctural equality condition with outcome $K_i \neq L_j$; d) creating sustainability gaps in the process ($SG > 0$); and therefore, e) this leads to structural falsification.

❖ Case B: Ki-driven system

If we apply unilateral pressure to K_i , we create a system led by K_i , which can be stated as $K_i \rightarrow L_j$. The characteristics of the K_i led system are a) K_i is the independent driver of L_j ; b) L_j becomes the dependent outcome; c) maximization of K_i ($MAX(K_i)$) breaks the system equality in the sense that it violates the conjunctural equality condition with outcome $K_i \neq L_j$; d) creating sustainability gaps in the process ($SG > 0$); and therefore, e) this leads to structural falsification.

3) Core structural implications: The moment a conjunctural system ($K_i \leftrightarrow L_j$) is represented as a directional system ($K_i \rightarrow L_j$ or $K_i \leftarrow L_j$), structural inconsistency is introduced; and this is independent of the specific paradigm.

V. THE NATURE OF THE TRUE SUSTAINABILITY INDEX FALSIFICATION

The idea of the general true system sustainability index framework (GTSSI) was introduced in practice in 2003 (Muñoz 2003) applied to agenda 21 indicators and the theory of true sustainability on which that article was based was formally published in 2009 (Muñoz 2009), and they defined the general true system sustainability index as:

$$GTSSI = P/N = \frac{P}{N}$$

Where P = Number of desirable characteristics present and N = Total number of desirable characteristics. And this idea was used to develop system specific indices and a general system sustainability index based on present(P)-absent(A) sustainability thinking, representing the society, the environment, and the economy as component specific indices; and reflecting society, environment and economy in the general index. Parallel to those lines the system specific indices for component Ki and Lj and for the general true sustainability index (TSSI) for Ki and Lj using present (P)/absent sustainability thinking can be stated as follows:

$$SSI_{Ki} = P_{Ki} / N_{Ki} \text{ for subsystem Ki}$$

$$SSI_{Lj} = P_{Lj} / N_{Lj} \text{ for subsystem Lj}$$

$$TSSI_{KiLj} = P_{Ki} + P_{Lj} / N_{Ki} + N_{Lj} \text{ for the general true sustainability index KiLj}$$

Where P_{Ki} , P_{Lj} , and $P_{Ki} + P_{Lj}$ = Number of desirable characteristics Ki, Lj and Ki +Lj present, respectively while N_{Ki} , N_{Lj} , and $N_{Ki} + N_{Lj}$ = Total number of desirable characteristics, respectively. The TSSI is the general true sustainability index for the system with components Ki and Lj.

The structure of all the formulas, general and system specific, above reflect interdependence, which means they are conjunctural in nature.

1) The two ways nature of structural consistency in true sustainability systems: The process of two ways sustainability index consistency can be represented in different forms, general and component specific as $P \leftrightarrow N$, $P_{Ki} \leftrightarrow N_{Ki}$, $P_{Lj} \leftrightarrow N_{Lj}$, $P_{Ki} + P_{Lj} \leftrightarrow N_{Ki} + N_{Lj}$. This means that this process is a) a bi-directional relationship; b) a joint determination; c) a balanced condition $GTSSI = P / N$, $SSI_{Ki} = P_{Ki} / N_{Ki}$, $SSI_{Lj} = P_{Lj} / N_{Lj}$, and $TSSI_{KiLj} = P_{Ki} + P_{Lj} / N_{Ki} + N_{Lj}$; d) a joint optimization type of adjustment with outcome $[(GTSSI)^* = (P / N)^*]$, $[(SSI_{Ki})^* = (P_{Ki} / N_{Ki})^*]$, $[(SSI_{Lj})^* = (P_{Lj} / N_{Lj})^*]$, and $[(TSSI_{KiLj})^* = (P_{Ki} + P_{Lj} / N_{Ki} + N_{Lj})^*]$; e) a process that leads to no sustainability gaps ($SG = 0$); and f) a process where structural consistency is maintained or restored. And therefore, the above is the only complete structurally admissible representation of a conjunctural sustainability index system using present/absence theory as far as I know. Notice that when systems are treated conjuncturally the numerator and the denominator are jointly determined and ratios stabilize coherently resulting in no sustainability gap creation; and therefore, there is no artificial inflation of performance.

2) The one way nature of sustainability index inconsistency in separable systems: If we assume or take conjunctural index systems $GTSSI = P / N$, $SSI_{Ki} = P_{Ki} / N_{Ki}$, $SSI_{Lj} = P_{Lj} / N_{Lj}$, and $TSSI_{KiLj} = P_{Ki} + P_{Lj} / N_{Ki} + N_{Lj}$ are separable, then we introduce structural inconsistencies when we apply unilateral pressures for example to either component P, P_{Ki} , P_{Lj} , $P_{Ki} + P_{Lj}$ as numerators or as denominators, as shown in the following cases

❖ Case A: Denominator-driven system for $GTSSI = P/N$

If we apply unilateral pressure to N, we create a system led by N, which can be stated as $P \leftarrow N$. The characteristics of the N led system are a) N is the independent driver of P; b) P becomes the dependent outcome; c) maximization of N ($MAX(N)$) breaks the system equality as it violates the conjunctural equality condition with outcome $GTSSI \neq P/N$; d) creating sustainability gaps in the process ($SG > 0$); and therefore, e) this leads to structural falsification. The same thinking applies to the other sustainability indices when under unilateral denominator pressures $P_{Ki} \leftarrow N_{Ki}$, $P_{Lj} \leftarrow N_{Lj}$, and $P_{Ki} + P_{Lj} \leftarrow N_{Ki} + N_{Lj}$.

❖ **Case B: Numerator driven system for GTSSI = P/N**

If we apply unilateral pressure to P, we create a system led by P, which can be stated as $P \rightarrow N$. The characteristics of the P led system are a) P is the independent driver of N; b) N becomes the dependent outcome; c) maximization of P (MAX (P)) breaks the system equality in the sense that it destabilizes the conjunctural equality condition with outcome $GTSSI \neq P/N$; d) creating sustainability gaps in the process ($SG > 0$); and therefore, e) this leads to structural falsification. The same thinking applies to the other sustainability indices when under unilateral numerator pressures $PK_i \rightarrow NK_i$, $PL_j \rightarrow NL_j$, and $PK_i + PL_j \rightarrow NK_i + NL_j$.

3) Core structural implications: The moment a denominator or the numerator of a conjunctural sustainability index system ($P \leftrightarrow N$, $PK_i \leftrightarrow NK_i$, $PL_j \leftrightarrow NL_j$, $PK_i + PL_j \leftrightarrow NK_i + NL_j$) is represented as a directional system ($P \rightarrow N$ or $P \leftarrow N$; $PK_i \leftarrow NK_i$ or $PK_i \rightarrow NK_i$; $PL_j \leftarrow NL_j$ or $PL_j \rightarrow NL_j$; $PK_i + PL_j \leftarrow NK_i + NL_j$ or $PK_i + PL_j \rightarrow NK_i + NL_j$) structural inconsistency is introduced; and this is independent of the specific sustainability index.

VI. STRUCTURAL SYSTEM FALSIFICATION AND SUSTAINABILITY INDEX SYSTEM FALSIFICATION EQUIVALENCIES

a) Conjunctural consistency equivalency: For the conjunctural system $K_i = L_j$ and the conjunctural sustainability index framework for K_i and L_j under present/absent conjunctural thinking $TSSIK_iL_j = PK_i + PL_j / NK_i + NL_j$ and when treated both as conjunctural systems there is a measurable equivalence to the structural consistency where the sustainability gap is zero ($SG = 0$) as indicated below:

$$K_i \leftrightarrow L_j \rightarrow SG = 0 \approx PK_i + PL_j \leftrightarrow NK_i + NL_j \rightarrow SG = 0$$

The expression above tells us that structural consistency is equivalent to sustainability index consistency, which means that the structural gap is being measured by the index gap, and when the measured gap is zero ($SG = 0$), then the model's structural validity is determined in theory and in practice.

b) Non-conjunctural inconsistent equivalency: For the conjunctural system $K_i = L_j$ and the conjunctural sustainability index framework for K_i and L_j under present/absent conjunctural thinking $TSSIK_iL_j = PK_i + PL_j / NK_i + NL_j$ and when treated both as separable systems there is a measurable equivalence to the structural failure where the sustainability gap is greater than zero ($SG > 0$) as indicated below:

$$[K_i \leftarrow L_j \text{ or } K_i \rightarrow L_j] \rightarrow SG > 0 \approx [PK_i + PL_j \leftarrow NK_i + NL_j \text{ or } PK_i + PL_j \rightarrow NK_i + NL_j] \rightarrow SG > 0$$

The expression above shows that structural inconsistency is equivalent to sustainability index inconsistency, which means that the structural gap is being measured by the index gap, and when the measured gap is greater than zero ($SG > 0$), then the model's structural invalidity is determined in theory and in practice.

VII. THE EQUIVALENCE OF THE STRUCTURAL FALSIFICATION RATIO AND THE TRUE SUSTAINABILITY INDEX RATIO

The structural index ratio (STIR) for K_i and L_j can be represented as dividing a specific case over the general case such that $STIR_{K_iL_j} = (K = L) / (K_i = L_j)$ and the true sustainability index ratio ($TSSIK_iL_j$) for K_i and L_j is $TSSIK_iL_j = PK_i + PL_j / NK_i + NL_j$; and then the following holds:

a) The structural falsification ratio: Therefore, the ratio to be structurally falsified is

$$STIR_{KiLj} = (K = L) / (Ki = Lj) = \frac{(K=L)}{(Ki=Lj)}$$

The expression above is the structural index ratio (STIR).

b) The true sustainability index ratio: The measurement of that structural index ratio is the true sustainability index ratio for Ki and Lj:

$$TSSIR_{KiLj} = P_{Ki} + P_{Lj} / N_{Ki} + N_{Lj} = \frac{PKi+PLj}{NKi+NLj}$$

The expression above is the true sustainability index ratio (TSSIR).

c) Linking the true sustainability ratio to optimality and distortions

i) The optimal true sustainability index ratio: If these equalities exist, $PKi = NKi$ and $PLj = NLj$, then optimality exists

$$[TSSIR_{KiLj}]^* = [P_{Ki} + P_{Lj} / N_{Ki} + N_{Lj}]^* = 1 \rightarrow SG = 0$$

ii) The distorted true sustainability index ratio: If these inequalities exist, $PKi \neq NKi$ and $PLj \neq NLj$, then distortions exist

$$TSSIR = P_{Ki} + P_{Lj} / N_{Ki} + N_{Lj} \neq 1 \rightarrow SG > 0$$

d) Linking the conjunctural structural ratio with optimality and distortions

i) The optimal conjunctural structural falsification ratio: If the structural falsification ratio ($STIR_{KiLj}$) is taken as a conjunctural condition, we arrive at the following optimal structure:

$$[STIR_{KiLj}]^* = [(K = L) / (Ki = Lj)]^* = (K = L)^* / (Ki = Lj)^* = 1 \rightarrow SG = 0$$

The above expression indicates that structural consistency is maintained or restored or exists so there is no sustainability gap creation ($SG = 0$) and there is validation.

ii) The distorted conjunctural structural falsification ratio: If the structural falsification ratio ($STIR_{KiLj}$) is taken as a separable condition, we arrive at the following structural inconsistency with system consistent optimality requirements (NSC^*), which creates sustainability gaps ($SG > 0$) and there is falsification.

$$[STIR_{KiLj}]^{nsc*} = [(K = L) / (Ki = Lj)]^{nsc*} = (K = L)^{nsc*} / (Ki = Lj)^{nsc*} \neq 1 \rightarrow SG > 0$$

The expression above indicates that structural consistency is broken or structural inconsistency exists so there is sustainability gap creation ($SG > 0$) and there is invalidation.

e) Structural validation and invalidation consistencies

i) Validation consistency: When we have a structural index ratio STIR treated as a conjunctural ratio then we have equivalence to the optimal true sustainability index measurement of consistency as indicated below:

$$STIR^* \approx TSSIR^* \rightarrow SG = 0$$

Optimal structural and measurement consistency emerge at the point where the sustainability gap is zero ($SG = 0$).

ii) Validation inconsistency: When we have a structural index ratio STIR treated as a separable ratio then we have equivalence to the true sustainability index measurement of inconsistency or distortion as indicated below, where sustainability gaps are created:

$$\text{STIR}^{\text{nsc}*} \approx \text{TSSIR} \rightarrow \text{SG} > 0$$

Structural and measurement inconsistency emerge at the point where the sustainability gap is greater than zero ($\text{SG} > 0$).

VIII. THE GENERAL STRUCTURAL-INDEX CONSISTENCY EQUIVALENCE CONDITION

The consistency equivalence condition linking structural falsification with sustainability index validation can be summarized as follows:

$$\text{Ki} \leftrightarrow \text{Lj} \Leftrightarrow \text{PKi} + \text{PLj} \leftrightarrow \text{NKi} + \text{NLj} \Leftrightarrow \text{TSSIR} = 1 \Leftrightarrow \text{SG} = 0$$

The expression above indicates that structural consistencies are equivalent to sustainability index consistencies, where no sustainability gaps are created or measured.

IX. THE GENERAL STRUCTURAL-INDEX INCONSISTENCY EQUIVALENCE CONDITION

The inconsistency equivalence condition linking structural falsification with sustainability index invalidation can be summarized as follows:

$$\text{Ki} \rightarrow \text{Lj} \text{ or } \text{Ki} \leftarrow \text{Lj} \Leftrightarrow \text{PKi} + \text{PLj} \rightarrow \text{NKi} + \text{NLj} \text{ or } \text{PKi} + \text{PLj} \leftarrow \text{NKi} + \text{NLj} \Leftrightarrow \text{TSSIR} \neq 1 \Leftrightarrow \text{SG} > 0.$$

Structural inconsistencies are equivalent to sustainability index inconsistencies, where sustainability gaps are created and measured.

X. MAIN IMPLICATIONS

i) Sustainability indices as structural projections: The true sustainability index is not an independent measurement construct. It is a numerical projection of the underlying structural condition of the system: i) When the system is structurally consistent ($\text{Ki} \leftrightarrow \text{Lj}$), the index converges to $\text{TSSIR}^* = 1 \rightarrow \text{SG} = 0$; and ii) When the system is structurally inconsistent ($\text{Ki} \rightarrow \text{Lj}$ or $\text{Ki} \leftarrow \text{Lj}$), the index reflects this distortion $\text{TSSIR} \neq 1 \rightarrow \text{SG} > 0$. Therefore, sustainability indices numerically express the presence or absence of structural consistency.

ii) Structural falsification vs. index invalidation: A fundamental distinction emerges: i) Structural falsification invalidates the model itself ($\text{Ki} = \text{Lj}$); and ii) Index invalidation reveals inconsistency in the measurement of that model. However, these are not independent processes. They are linked through a one-to-one mapping: Every structural inconsistency generates a corresponding index distortion.

iii) One-to-one equivalence principle: The paper establishes the following equivalence: Structural gap ($\text{SG} > 0$) $\leftrightarrow$ Index gap ($\text{TSSIR} \neq 1$). And this implies that there exists a bijective relationship between structural falsification and index invalidation. Hence, indices do not merely detect problems—they mirror them.

iv) Critical implication for empirical research: A system may appear empirically valid through well-constructed indicators while being structurally inconsistent. Therefore, empirical validity does not guarantee structural validity. And this establishes the need for structural validity as a pre-empirical condition followed by index-based measurement as a secondary validation layer.

v) Unified falsification framework: By linking the structural falsification ($K_i = L_j$) framework and the true sustainability index invalidation (TSSI framework) the paper provides a unified framework where structural theory and empirical measurement become analytically inseparable.

XI. CONCLUSIONS

First, it was pointed out that non-distorted conjunctural systems like $K_i = L_j$ have structural consistency; and when those non-distorted conjunctural systems like $K_i = L_j$ are taken as separable, then inconsistencies are introduced structurally falsifying the model $K_i = L_j$ in the process. Second, it was shown that when non-distorted systems are interpreted the true sustainability index provides a numerical representation of structural consistency and when distorted systems are analyzed the true sustainability index provides a numerical representation of the structural flaws or gaps. Third, it was highlighted that structural falsification invalidates the framework / model $K_i = L_j$ itself while the sustainability index invalidates the measurement of a system under that framework $K_i = L_j$. Fourth, it was stressed that a general implication of the ideas shared above is that an index can look —well-constructed— empirically, yet be structurally invalid. Fifth, it was indicated that a one-to-one correspondence between structural falsification and index invalidation exists, which implies that measurement (index) is structurally determined—not independent, mapping that way conjunctural structural reality (theory) with conjunctural empirical sustainability index tools(practice), respecting then the theory-practice consistency principle, which means i) metrics inherit ontology, ii) indicators cannot escape model structure, iii) empirical neutrality is impossible under structurally distorted systems; and therefore, iv) empirical coherence cannot rescue structural incoherence as empirical incoherence reflects structural incoherence. Overall, it was shown that structural inconsistency $\Leftrightarrow$ index inconsistency.

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